Pseudospectral Simulation of Shock-Turbulence Interactions

John V. Shebalin*
NASA Langley Research Center, Hampton, Virginia 23681

Introduction

In this Note, certain aspects of the direct numerical simulation of those highly compressible turbulent flows that can be described by the single-fluid Navier-Stokes equations within the constraint of periodic boundary conditions will be discussed. Although modern theoretical analyses of compressible turbulent flows began with the work of Moyal,[1] their direct numerical simulation, by a variety of methods, is much more recent. Here, we will only be concerned with pseudospectral Fourier methods; following the seminal work of Passot and Pouquet,[2] many other authors have also used these methods, for example, Erlebacher et al.,[3] Blaisdell et al.,[4] Sarkar et al.,[5] Kida and Orszag,[6] and Zang et al.[7]

These methods can be extended in two ways. First, we describe a technique that has proven useful previously[8]: the governing equations are cast in a form where the important physical variables are not the fluid density and temperature directly but rather their natural logarithms; this ensures adherence to a physical constraint of positive definiteness that may be computationally violated in nonlogarithmic formulations, leading to numerical instability.[9] Second, and of greater novelty, bulk viscosity[10] is utilized, both to model polyatomic gases more accurately and also to ensure numerical stability and accuracy in the presence of strong shocks.

Basic Equations

Here we will consider polytropic gases, i.e., gases such that the specific heats and their ratio $\gamma = c_p/c_v$ are taken to be constants. In addition, the viscosity μ, bulk viscosity μ_B, and thermal conductivity κ will also be assumed to be constant. Placing $\rho = \rho_0 e^{\lambda}$ and $T = T_0 e^{\sigma}$ into the gasdynamic equations[11] yields the basic nondimensional equations in a logarithmic formulation:

$$\frac{\partial \lambda}{\partial t} + \boldsymbol{u}\cdot\nabla\lambda = -\nabla\cdot\boldsymbol{u} \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\frac{1}{\gamma}e^{\sigma}\nabla(\lambda+\sigma) + \mu e^{-\lambda}\nabla \times\left[\nabla\boldsymbol{u} + \left(\frac{1}{3}+\beta\right)\boldsymbol{I}\nabla\cdot\boldsymbol{u}\right] \tag{2}$$

$$\frac{\partial \sigma}{\partial t} + \boldsymbol{u}\cdot\nabla\sigma = -(\gamma-1)\nabla\cdot\boldsymbol{u} + \kappa e^{-\lambda}[\nabla^2\sigma + (\nabla\sigma^2)] + \gamma(\gamma-1)\mu e^{-(\lambda+\sigma)}\left[\frac{1}{2}\tau'_{ij}\tau_{ij} + \beta(\nabla\cdot\boldsymbol{u})^2\right] \tag{3}$$

where, $\tau_{ij} = \partial_i u_j + \partial_j u_i - 2/3\delta_{ij}\nabla\cdot\boldsymbol{u}$ and $\boldsymbol{I} = [\delta_{ij}]$ is the unit dyadic. Also, $\beta = \mu_B/\mu$ is the ratio of bulk to shear viscosity. Logarithmic formulations are not uncommon and are useful in various contexts, e.g., Ref. 12.

Once $\mu = 1/Re$ is set, the other transport coefficients follow somewhat directly. Since the Prandtl number $Pr = \gamma\kappa/\mu \approx 1$ and since $1 < \gamma \le 5/3$ for common gases, we see $\kappa \approx \mu$. The value of μ_B, or equivalently $\beta = \mu_B/\mu$, is highly dependent on whether the gas is polyatomic or not; although monatomic gases have $\beta \approx 0$, polyatomic gases range from $\beta \approx 1$ for air to $\beta \approx 30$ for molecular hydrogen to $\beta \approx 10^3$ for carbon dioxide.[13]

Numerical Method

A Fourier pseudospectral technique[14] is used to numerically solve Eqs. (1–3). In this method, the physical variables λ, $\boldsymbol{u}$, and σ are expanded in terms of discrete Fourier series, and a third-order time-integration method is used[15]; again, details are presented elsewhere.[11]

The pseudospectral method, as used here, allows the shock structure to be resolved. This is done by considering cases where $\beta = \mu_B/\mu$ is large enough so that whatever shocks occur are of a naturally limited steepness[16]; for example, the gas under consideration can be H_2 (molecular hydrogen) or some mixture containing H_2, or the characteristic length L_0 can be assumed to be small for an arbitrary polyatomic gas. In contrast, numerical solutions of the Euler equations require methods that use shock capturing or shock fitting.[14]

To begin a simulation, the initial values of λ, $\boldsymbol{u}$, and σ need to be specified (but not the boundary conditions, which are periodic). The initial turbulent velocity is set according to

$$|\boldsymbol{u}(\boldsymbol{k})|^2 \sim k^4 \exp(-2k^2/k_0^2) \tag{4}$$

where k_0 is the wave number at which the spectrum peaks; the phase of the $\boldsymbol{u}(\boldsymbol{k})$ is initially random. In setting the initial conditions on the velocity, it is useful to decompose it into "incompressible" and "compressible" parts; in terms of Fourier coefficients, the decomposition is easily affected:

$$\boldsymbol{u}(\boldsymbol{k}) = \boldsymbol{u}^s(\boldsymbol{k}) + \boldsymbol{u}^c(\boldsymbol{k}) \tag{5}$$

where the solenoidal (i.e., incompressible) part $\boldsymbol{u}^s$ and compressible part $\boldsymbol{u}^c$ are

$$\boldsymbol{u}^s(\boldsymbol{k}) = (\boldsymbol{I} - \hat{\boldsymbol{k}}\hat{\boldsymbol{k}})\cdot\boldsymbol{u}(\boldsymbol{k}), \qquad \boldsymbol{u}^c(\boldsymbol{k}) = \hat{\boldsymbol{k}}\hat{\boldsymbol{k}}\cdot\boldsymbol{u}(\boldsymbol{k}) \tag{6}$$

and where $\hat{\boldsymbol{k}}$ is the unit vector in the direction of $\boldsymbol{k}$. The $\boldsymbol{k} = 0$ component of $\boldsymbol{u}^s$ corresponds to the mean flow velocity.

For the moment assume that we have an initially incompressible flow. Then $\boldsymbol{u}^c = 0$ and $\rho = 1$ at $t = 0$; upon taking the divergence of Eq. (2) and defining $T = 1 + T'$, we find that the fluctuation T' initially satisfies:

$$-\nabla^2 T' = \gamma\partial_i\partial_j u_i^s u_j^s = S(\boldsymbol{x}) \tag{7}$$

In terms of Fourier coefficients, the temperature fluctuations that are consistent with the assumption of an initially incompressible flow are given by

$$T'(\boldsymbol{k}) = k^{-2}S(\boldsymbol{k}) \tag{8}$$

where $k > 0$. Since $T'(\boldsymbol{x}) > -1$ so that $T > 0$, and since $T' \sim \gamma|\boldsymbol{u}^s|^2$, then $\boldsymbol{u}^s$ must be scaled accordingly. The mean value of $|\boldsymbol{u}^s|^2$ (denoted by $\langle|\boldsymbol{u}^s|^2\rangle$) is the initial turbulent Mach number squared M_t^2. The maximum fluctuation of $|\boldsymbol{u}^s|^2$ is larger than M_t^2, and we expect that max $M_t \sim \gamma^{-1}$ in order that the constraint $T' > -1$ is obeyed.

Numerical Results

The pseudospectral logarithmic variable method was implemented in both a two-dimensional code and a three-dimensional code. The two-dimensional code was used to examine the passage of a region of turbulent flow through a shock on a 512^2 grid, whereas the three-dimensional code was used to simulate supersonic isotropic turbulence on a 64^3 grid. There were no problems experienced in any of these simulations (such as conservation of

Received Jan. 9, 1993; revision received April 1, 1993; accepted for publication April 7, 1993.

*Senior Research Scientist, Theoretical Flow Physics Branch; currently Visiting Scientist, Institute for Computer Applications in Science and Engineering (ICASE). Member AIAA.

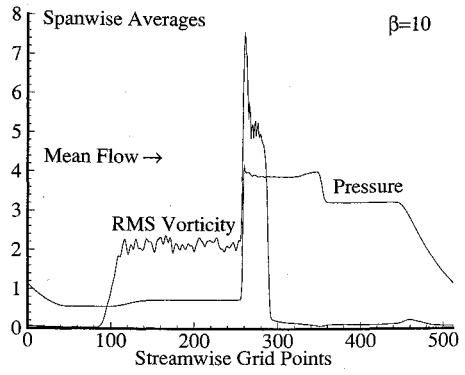

Fig. 1 Spanwise averages for pressure and rms vorticity for run 2D512A ($\beta = 10$) at $t = 0.567$.

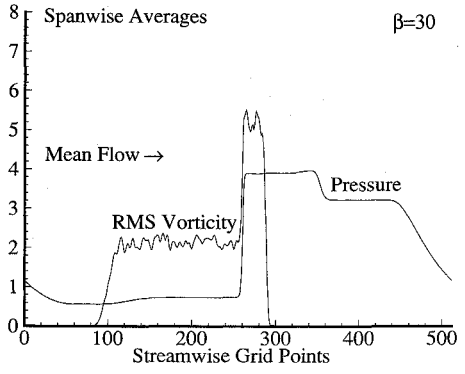

Fig. 2 Spanwise averages for pressure and rms vorticity for run 2D512B ($\beta = 30$) at $t = 0.567$.

energy) as long as the dissipation wave number k_D began and remained less than k_{max}.[11] Here, only the two-dimensional results will be discussed.

Consider three of these runs—2D512A, 2D512B, and 2D512C—that differed initially only in the value of β assigned to each: 10, 30, and 100, respectively; for all three runs, $\mu = 0.001$, $\kappa = 0.001$, and $\gamma = 1.4$. The speed of the code was about 6 CPU-s/Δt on a Cray Y-MP; the number of time steps required to run from $t = 0$ to $t = 0.567$ were 5350 for $\beta = 10$, 5650 for $\beta = 30$, and 15,560 for $\beta = 100$. All of the runs began with identical initial conditions, which consisted of a region of incompressible turbulence satisfying Eq. (4) with $k_0 = 9$ and $M_t = 0.2$ filling the left half of the two-dimensional grid and a shock wave in the right half of the grid. Using the shock jump conditions,[17] density, temperature, and velocity corresponding to a Mach 2 shock were set inside the right half of the grid, whereas in the left half of the grid the mean density was $\langle \rho \rangle = 1$ and the mean velocity was $\langle u_x \rangle = 2$ (the frame of reference was such that the shock front was initially stationary and the turbulent region was moving from left to right into it). At the edges of the turbulent region, there were transition regions of 20 grid points in the x direction (i.e., the streamwise direction) in which the interior turbulent velocity field went smoothly to zero.[18] The turbulent temperature fluctuations were determined by Eq. (8).

Similarly, there was a smooth transition from the freestream values of density, temperature, and velocity to their jump values, and back again, over the 20 grid points on the outside edges of the shocked region. Thus, we begin on the leftmost edge of the grid with freestream values, make a transition into a turbulent region, make another transition out of the turbulent region back into the freestream values, and then make a transition into the shocked region, followed by a transition back to freestream values on the rightmost edge of the grid. This resulted in a periodic set of initial conditions that could be treated by a Fourier method. These initial conditions can be thought of as 1) a one-dimensional Mach 2 shock wave and 2) a localized region of eddy turbulence placed into the freestream flow (and moving with it) just ahead of the shock front.

Once evolution began, the front of the shock steepened slightly, and the density and temperature increased slightly, leading to a pressure jump ratio of 5 across the shock front (it was initially 4.5). Based on this pressure ratio, the Mach number of the shock should have risen to 2.1; the shock front, which would have remained stationary in the reference frame chosen, was observed to move forward at a relative velocity of 0.1, in accordance with expectations. During the same time, the back of the initial shock pulse spread into a rarefaction wave. In these two-dimensional runs, k_D started and remained $< k_{max} = 256$, so that the turbulence was sufficiently well resolved.

The resolution of the shock front, in turn, gets better with increasing values of β. (A simulation with $\beta = 1$ was attempted, but the shock quickly steepened beyond the resolution of the code, resulting in energy nonconservation, although there was no blowup.) Consider Figs. 1 and 2, where spanwise averages for vorticity and pressure are shown for the two runs for the same time ($t = 0.567$). In Fig. 1, which corresponds to $\beta = 10$, there are some oscillations in the pressure at the shock front; these oscillations disappear, however, in Fig. 2, which corresponds to $\beta = 30$ (the $\beta = 100$ case looked very similar).

In Fig. 2, the vorticity jumps by a factor of about 2.5, which is also the case for Fig. 1, if the initial overshoot is ignored. This is consistent with linear theory.[19,20] The shape of the shocks is also consistent with the results from previous simulations using essentially nonoscillatory methods.[21]

Conclusion

In this paper, two extensions of pseudospectral Fourier methods for solving the compressible Navier-Stokes equations have been described. The first involved a "logarithmic variable" formulation of the basic equations, and the second involved using realistic values of the ratio of bulk viscosity to shear viscosity. It was demonstrated that shock structure could be resolved and shock-turbulence interactions could be examined by direct numerical simulation.

The novel feature of the work presented here is the demonstration that bulk viscosity can be used to resolve shock structure. This is useful for realistically modeling shocks in such gases as molecular hydrogen and carbon dioxide. However, even in the case of monatomic gases that have small or negligible bulk viscosities, a large numerical value can serve as an "artificial" bulk viscosity, for the same reasons that an artificial shear viscosity was introduced into finite difference methods many years ago.[22] An artificial bulk viscosity, however, has the benefit that it does not affect the incompressible part of the flow.

Acknowledgments

I would like to thank G. Erlebacher, M. Y. Hussaini, S. Sarkar, and C. A. Kennedy for many useful discussions. I would also like to thank M. Y. Hussaini for the opportunity to spend a year in residence at ICASE as a visiting scientist.

References

[1]Moyal, J. E., "The Spectra of Turbulence in a Compressible Fluid; Eddy Turbulence and Random Noise," *Mathematical Proceedings of the Cambridge Philosophical Society*, Vol. 48, No. 1, 1952, pp. 329–344.

[2]Passot, J., and Pouquet, A., "Numerical Simulation of Compressible Homogeneous Flows in the Turbulent Regime," *Journal of Fluid Mechanics*, Vol. 181, Aug. 1987, pp. 441–446.

[3]Erlebacher, G., Hussaini, M. Y., Kreiss, H. O., and Sarkar, S., "The Analysis and Simulation of Compressible Turbulence," *Theoretical and Computational Fluid Dynamics*, Vol. 2, No. 2, 1990, pp. 73–95.

[4]Blaisdell, G. A., Mansour, N. N., and Reynolds, W. C., "Numerical Simulations of Compressible Homogeneous Turbulence," Stanford Univ., Rept. No. TF-50, Stanford, CA, 1991.

[5]Sarkar, S., Erlebacher, G., and Hussaini, M. Y., "Direct Simulation of Compressible Turbulence in a Shear Flow," *Theoretical and Computational Fluid Dynamics*, Vol. 2, No. 5, 1991, pp. 291–305.

[6]Kida, S., and Orszag, S., "Energy and Spectral Dynamics in Decaying Compressible Turbulence," *Journal of Scientific Computation*, Vol. 7, No.

1, 1992, pp. 1–34.
[7]Zang, T. A., Dahlburg, R. B., and Dahlburg, J. P., "Direct and Large Eddy Simulations of Three-Dimensional Compressible Turbulence," *Physics of Fluids A*, Vol. 4, No. 1, 1992, pp. 127–140.
[8]Shebalin, J. V., and Montgomery, D., "Turbulent Magnetohydrodynamic Density Fluctuations," *Journal of Plasma Physics*, Vol. 39, No. 2, 1988, pp. 339-367.
[9]Zeman, O., "Progress in Modeling Hypersonic Turbulent Boundary Layers," *Annual Research Briefs–1992*, Center for Turbulence Research, NASA Ames Research Center, Moffett Field, CA, Jan. 1993, pp. 213–225.
[10]Vincenti, W. G., and Kruger, C. H., *Introduction to Physical Gas Dynamics*, Wiley, New York, 1965, pp. 407–412.
[11]Shebalin, J. V., "Pseudospectral Simulation of Compressible Turbulence Using Logarithmic Variables," AIAA 11th Computational Fluid Dynamics Conference, AIAA Paper 93-3375, Orlando, FL, July 1993.
[12]Abarbanel, S., Dutt, P., and Gottlieb, D., "Splitting Methods for Low Mach Number Euler and Navier-Stokes Equations," *Computers & Fluids*, Vol. 17, No. 1, 1989, pp. 1–12.
[13]Thompson, P. A., *Compressible-Fluid Dynamics*, McGraw-Hill, New York, 1972, p. 30.
[14]Canuto, C., Hussaini, M. Y., Quateroni, A., and Zang, T. A., *Spectral Methods in Fluid Dynamics*, Springer-Verlag, New York, 1988, p. 83, pp. 255–273.
[15]Gazdag, J., "Time-Differencing Schemes and Transform Methods," *Journal of Computational Physics*, Vol. 20, No. 2, 1976, pp. 196–207.
[16]Gilbarg D., and Paolucci, D., "The Structure of Shock Waves in the Continuum Theory of Fluids," *Journal for Rational Mechanics and Analysis*, Vol. 2, No. 4, 1953, pp. 617–642.
[17]Landau, L. D., and Lifshitz, E. M., *Fluid Mechanics*, 2nd ed., Pergamon, Oxford, England, UK, 1987, p. 335.
[18]Shebalin, J. V., "Aerobrake Plasmadynamics: Macroscopic Effects," *Journal of Spacecraft and Rockets*, Vol. 28, No. 4, 1991, pp. 394–400.
[19]Ribner, H. S., "Spectra of Noise and Amplified Turbulence Emanating from Shock-Turbulence Interaction," *AIAA Journal*, Vol. 25, No. 3, 1987, pp. 436–442.
[20]Lee, S., Lele, S. K., and Moin, P., "Direct Numerical Simulation of Shock Turbulence Interaction," AIAA Paper 91-0523, Jan. 1991.
[21]Hannappel, R., and Friedrich, R., "Interaction of Isotropic Turbulence with a Normal Shock Wave," *Applied Scientific Research*, Vol. 51, No. 2, 1993, pp. 507–512.
[22]Richtmeyer, R. D., and Morton, K. W., *Difference Methods for Initial Value Problems*, 2nd ed., Interscience, New York, 1967, p. 312.

Generalized Vortex Lattice Method for Oscillating Thin Airfoil in Subsonic Flow

Paulo A. O. Soviero*
Instituto Tecnológico de Aeronáutica, 12228-900 São José dos Campos, S.P., Brazil

Nomenclature

h = nondimensional vertical displacement amplitude
J_0 = Bessel's function of first kind and zero order
K = $krM/(1-M^2)$
kr = reduced frequency
M = undisturbed flow Mach number
Y_0 = Besssel's function of second kind and zero order
β = $(1-M)^{1/2}$
$\delta\phi$ = perturbation velocity potential jump across airfoil and wake
πk = complex lift coefficient for unitary amplitude
$\pi m/2$ = complex pitch moment coefficient, about midchord, for unitary amplitude

Superscripts
$'$ = real part of k and m
$''$ = imaginary part of k and m

Subscripts
a = relative to heaving motion
b = relative to pitching motion

Received Feb. 26, 1992; revision received Aug. 10, 1992; accepted for publication Aug. 12, 1992.
*Professor, Aeronautical Engineering Division. Member AIAA.

Introduction

DESPITE the wide attention paid to the important problem of a harmonically oscillating thin airfoil in a subsonic flow,[1,2] the panel method approach,[3] or even its simplest variant, namely, the vortex lattice method, has been used only recently for the problem solution. The reason for this may be attributed to the fact that, when solving the problem for the velocity potential, the wake must explicitly be taken in account. Another reason may be the already wide accumulation of knowledge of singular kernels which occurs when acceleration potential formulations are made, as in Possio's integral equation.[2]

Except for approximated solutions,[4] it appears that numerical methods are the only tools which may be employed to solve the high frequency and Mach number cases without significant losses in precision and in a sufficiently general way.

In the present work, the generalized vortex lattice method[5] is applied to the two-dimensional case of harmonically oscillating flat plate in a subsonic flow. Two modes of motion have been considered, namely, the pitching and heaving modes.

The reasons the present method is called a generalized vortex lattice will be presented, qualitatively, in the next section. For a complete discussion about the method the reader is referred to Ref. 5.

Mathematical Model and Its Integral Equation

A two-dimensional flat plate in small pitching and heaving harmonic motion is considered in an otherwise steady subsonic flow which streams along the positive X-axis direction.

The undisturbed flow velocity and reference length (semichord) are made unitary. In this case the linearized equation and boundary conditions for the perturbation velocity potential can be reduced to the following system of equations[5,6]:

$$\frac{\partial^2\phi}{\partial X^2}+\frac{\partial^2\phi}{\partial Z^2}+K^2\phi=0 \tag{1}$$

$$\frac{\partial\phi}{\partial Z}=\exp(-iKMX)/\beta\left(\frac{\partial h}{\partial X}+ikrh\right) \qquad -1\le X\le 1 \tag{2a}$$

$$\frac{iK}{M}\delta\phi+\frac{\partial\delta\phi}{\partial X}=0, \qquad X\ge 1 \tag{2b}$$

Equation (2a) is applied on flat plate surface and Eq. (2b) over the wake.

Solving Eq. (1) for ϕ, the nondimensional pressure coefficient for profile and wake is obtained from

$$C_p=-2\exp(iKMX)\left(\frac{iK}{M}\phi+\frac{\partial\phi}{\partial X}\right) \tag{3}$$

It is worthwhile to note that Eq. (2b) results from the application of Eq. (3) to both sides of the wake and then imposing pressure continuity. At the trailing edge, $X=1$, Eq. (2b) also assures the Kutta condition.

The integral equation that relates the velocity potential jump across the flat plate and wake to the velocity potential in an arbitrary field point can be expressed as[7]

$$\phi_P=-\frac{1}{4}\int_{-1}^{\infty}\delta\phi\,\frac{\partial}{\partial n}\,[D_0(KR)]\,dX \tag{4}$$